Lesson 7-7: Area of Circles, Sectors and Segments

Area of a sector or segment???

Hmm, that seems a bit tough. I mean, a polygon is one thing: all of its sides are straight line segments. It isn't tough to realize a polygon can be broken into simpler shapes (triangles, etc.) that we can easily handle. But…the area of a sector or segment? That's a bit tougher.

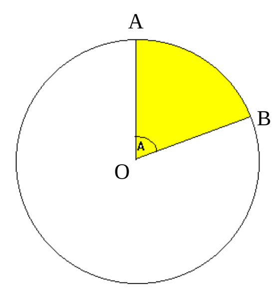
Lets start with something we know

I'll bet all of you can tell me how to find the area of a circle. If you can't off the top of your head, you'd be able to very quickly by looking it up. The area of a circle is πr^2 where *r* is the radius of the circle. This is a theorem…

Theorem 7-15 Area of a Circle

The **area of a circle** is the product of π and the square of the radius: $A = \pi r^2$

What exactly is a sector?



To define what these are we will use something we learned about yesterday: arcs.

A **sector of a circle** is a region between two radii of a circle and the included arc. In this picture, the yellow region is a sector. We name sectors by their center angle: in this case sector *AOB*.

Area of a sector

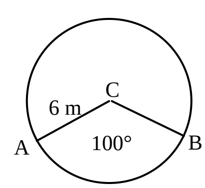
This seems tricky, but if you stop and think about it, it is really rather simple. The problem is very similar to one from yesterday…how to determine the length of an arc. If you recall, the length of an arc is a percentage of the circle's circumference.

Looking at the picture of a sector above, let's assume the measurement of the arc is 60. That means that the sector consumes 60% of the full circle right? So if we knew the area of the circle we could figure out the area of the sector. This leads us to…

Theorem 7-16 Area of a Sector of a Circle

The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle. The area of sector *AOB* = $\frac{m\overset{\frown}{AB}}{360} \bullet \pi r^2$

Area of a sector example



Find the area of sector *ACB*. Leave your answer in terms of π.

$$m\overset{\frown}{AB} = 100; \quad area \odot C = \pi(6)^2 = 36\pi$$

$$\text{Area of sector } AOB = \frac{m\overset{\frown}{AB}}{360} \bullet \pi r^2 = \frac{100}{360} \bullet 36\pi = \frac{100}{10} \bullet \pi = 10\pi \, m^2$$

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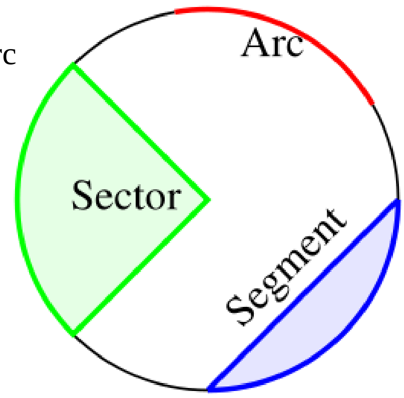
OK, what's a segment?

A **segment of a circle** is the part of a sector between the arc and a segment joining its endpoints.

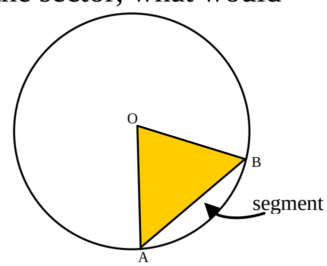


This picture shows all three of the new terms we're working with: arc, sector and segment. The segment is the blue shaded region.

Now, I'll bet you can guess what I'm going to ask next…

Area of a segment

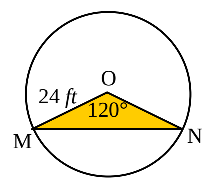
Yup…how do we determine the area of a segment? Well, we can start by recalling that a segment is a part of a sector. So, if we took the segment part out of the sector, what would be left? A triangle! Consider the following figure with sector *AOB*. The yellow triangle is what remains of the sector if you remove segment *AB*. Using this information we could say:



Area of segment = area of sector – area of triangle

Area of a segment example

Find the area of the segment *MON*. Round your answer to the nearest tenth.

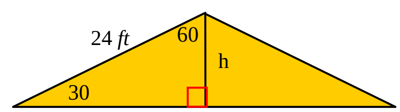


Area of sector: $\frac{m\widehat{MN}}{360} \bullet \pi r^2 = \frac{120}{360} \bullet \pi 24^2 = \frac{1}{3} \bullet \pi \bullet 24 \bullet 24 = 8 \bullet 24 \bullet \pi = 192\pi$

Area of triangle: 30-60-90 with h the short leg and the long leg ½ the base:

$$h = 24 \div 2 = 12; \quad b = 2 \bullet 12\sqrt{3} = 24\sqrt{3}; \quad A = \frac{1}{2}bh = \frac{1}{2} \bullet 12 \bullet 24\sqrt{3} = 144\sqrt{3}$$

<u>Area of segment</u>: $192\pi - 144\sqrt{3} \approx 353.77 \approx 353.8\ ft^2$



Homework Assignment

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